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NON-EUCLIDEAN GEOMETRY: HISTORICAL AND EXPOSITORY.

By GEORGE BRUCE HALSTED, A. M., (Princeton), Ph.D., (Johns Hopkins); Member of the London Mathematical Society; and Professor of Mathematics in the University of Texas, Austin, Texas.

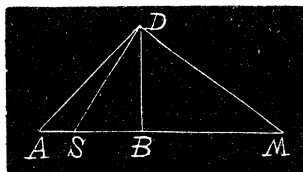
[Continued from the October Number.]

PROPOSITION X.

If the straight DB stands at right angles (fig. 8.) to a certain ABM , and the join DM be greater than the join DA , also the base BM will be greater than the base BA .

And inversely.

PROOF. And in the first place assuredly these bases will not be mutually equal. Otherwise (Eu. I. 4.) AD and DM would be equal, contrary



to the hypothesis. But neither will BA be greater than BM . Otherwise, in BA the portion BS being taken equal to BM , and SD joined, the angles BSD , BMD (Eu. I. 4.) would be equal: But the angle BSD is (Eu. I. 16.) greater than the angle BAD . Therefore the angle BMD would be greater than this.

But this is contrary to Eu. I. 18; since the side DM in the triangle MDA is supposed greater than the side DA .

It remains therefore, that the base BM is greater than the base BA .

Quod erat primo loco demonstrandum.

Next if either base, as BA suppose (the figure need not be changed) is conceived as greater than the other BM ; then the join DS , which cuts off from BA the portion SB equal to BM , will be equal (Eu. I. 4.) to the join DM .

Again the angle DSA will be obtuse (Eu. I. 16.) and the angle DAS acute (Eu. I. 17.)

Wherefore (Eu. I. 18.) the join DA will be greater than the join DS , and the join supposed equal to it DM . Quod erat secundo loco demonstrandum.

Itaque constant proposita.

PROPOSITION XI.

Let the straight AP (as long as you choose) cut the two straight PL , AD (fig. 9.) the first indeed at right angles in P , but the latter at A in any acute angle converging toward PL . I say the straight AD , PL (in the hypothesis of right angle) will at length, meet in some point, and indeed at a finite or terminated distance, if they are prolonged toward that side on which they make with the transversal AP two angles together less than two right angles.

PROOF. Prolong DA toward the other side to some point X , and through A erect to AP the perpendicular HAC , the point H being on the side of the angle XAP .

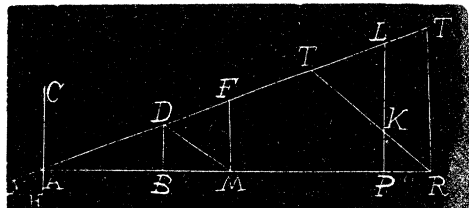
Then in AD produced toward the side of PL assume two equal inter-

vals AD , DF , and let fall upon AP the perpendiculars DB , FM , which assuredly fall (Eu. I. 17.) on the side of the acute angle DAP , and join DM .

I ought to show that the join DM will be equal to DF , or DA .

And in the first place indeed DM cannot be greater than DF . For otherwise the angle DMF would be less (Eu. I. 18.) than the angle DFM , or its equal (P.VIII, in the hypothesis of right angle) the angle XAH , or its vertical CAD .

Wherefore (since the angles CAM , FMA are assumed equal, as being right) the remaining angle DMA would be greater than the remaining angle DAM . But this is absurd (against Eu. I. 18.) if indeed DM be taken greater than DF , or DA .



But neither will DM be less than this DF . Otherwise the angle DMF would be greater (Eu. I. 18.) than the angle DFM , or its equal (P.VIII, in the hypothesis of right angle) the angle XAH , or its vertical CAD . Wherefore again, as above, the remaining angle DMA will not be greater, but less than the remaining angle DAM . But this is absurd (against Eu. I. 18.) since assuredly DM is taken less than DF , or DA .

It remains therefore, that the join DM is equal to DF , or DA . Wherefore in the triangle DAM (Eu. I. 5.) the angles at the points A , and M will be equal; and therefore in the triangles DBA , DBM , right-angled at B , the bases AB , BM will be equal (Eu. I. 26.).

This indeed was here our aim.

Since therefore, (assuming in AD produced the interval AF double the interval AD) the perpendicular FM dropped on the transversal AP cuts off from AP toward P a base AM double AB ; which the perpendicular let fall from the point D cuts off; it is manifest that this duplication of the preceding interval can be so many times repeated, that thus in AD continued we attain to a certain point T , from which the perpendicular let fall upon AP prolonged cuts off a certain AR greater than the finite AP however great.

But it holds, that this cannot happen, unless after the meeting of this prolonged AD with PL in some point L .

For if the point T occurred before this meeting, the perpendicular TR must cut PL in some point K . But then in the triangle KPR would be found two right angles at the points P , and R ; which is absurd (against Eu. I. 17.).

Therefore it holds that the straights AD , PL meet each other mutually (in the hypothesis of right angle) in some point (and indeed at a finite, or terminated distance) if they be produced toward that side, on which with the transversal AP (of finite length however great) they make two angles together less than two right angles. Quod erat demonstrandum.